

ASSESSMENT OF FORM TOLERANCES BY LEAST SQUARE METHOD

Gopinath L, Ravishankar S*
Centre for Civil Aircraft Design & Development,
CSIR-National Aerospace Laboratories, Bengaluru, India.
E-mail: lgopinath@nal.res.in, ravishankar@nal.res.in

Nagarajan S
Sri Krishna Institute of Technology, Chennai, India.
E-mail: snr1485@gmail.com

Abstract

Form tolerances are related to features that are not dependent on datum for reference, where the overall feature accomplishes form. Evaluation of form tolerance is done from the sample space data set, its distribution and measurement factors. Average value of deviation to assess the tolerance may give a quick picture of variation but shall not contribute to the characteristics of slope/fluctuations in the readings. The knowledge of the expected geometry achieved by best fit computation through any of the mathematical procedure provides the primary iterative step to define manufacturing variation. This paper discusses the best fit by Least Square Method by analyzing the data and deviations considered for the form tolerances such as Flatness, Circularity and Straightness. The standard mathematical definition for the validation of the form tolerances are drawn from the ASME Y14.5M standards. The case studies to evaluate the mathematical method are carried out for flatness on the surface plates, Circularity on Ring gauges and Straightness on a straight edge. The deviations of the points from the Gaussian geometry are compared against hard inspection methods and the reliability of the best fit by least square method is discussed against the BIS standards and its characteristics by a normal distribution curve.

Key words : Form tolerance, Least Square Method, Flatness, Circularity, Straightness, Hard inspection.

1. Introduction

Geometric Dimensioning & Tolerancing (GD&T) is a precise mathematical language that describes the design, dimensions, size, form, orientation and location of part feature.

*Corresponding Author

The American National Standards Institute publication in 1982 of ANSI Y14.5M-1982 was in the rigorous, unambiguous standardization of the methodology. Tolerance is defined as the magnitude of permissible variation of a dimension or other measured or control criterion from the specified value. Tolerances have to be allowed because of the inevitable human failings and machine limitations which prevent achieving nominal values during fabrication. The primary purpose of tolerances is to permit variation in dimensions without degradation of the performance where functional requirements will be the dominating factor in setting tolerances.

Form tolerances are applicable to single/individual features or elements that are not dependent on datum for reference. The form tolerance accommodates the following features like Flatness, Straightness, Circularity and Cylindricity of a part. These features fit with reliability by adopting best fit approach. The best fit can be achieved from several mathematical methods/procedures. Algorithms such as

iterative minimum acceptable duration zone localization algorithms are built to address challenges of frequent variation due to customization and complexity of parts [1]. An iterative reweighed least squares algorithm for form tolerance evaluation by updating the weighted coefficients iteratively was also reported [2]. Three theorems were proposed on the evaluation of both straightness and flatness for large number of points. The first theorem identifies the redundant data points; the second one explains the procedure to obtain the optimum solution by subset of data points. On the failure of second theorem, third theorem functions as a way to identify critical data points and update the subset to reach the optimum criterion [3]. Desired results are obtained when initial estimates of the variables are obtained using Least squares method, which gives the starting point for the linear approximation technique and this does not result in larger tolerance values and the function is minimized [4]. Algorithms for evaluating form tolerance using the orthogonal arrays and experimental optimization technique yielded results very much close to the minimum tolerance zones calculated using least square method. This was illustrated in a convex hull calculation [5]. Downhill simplex method and the repetitive bracketing method with the convergence criteria is considered for the evaluation of minimum zone flatness [6]. Form errors are computed using the linear deviations and simplex search method. This method gives smaller peak-to-valley values when compared to the least square method [7,8]. It explains the linear and normal deviations, using

the least square and minimum deviation techniques. The analysis result for the evaluation of surface by least square method gives deterministic solution and does not lead to minimum zone deviation. Among the various techniques, Monte Carlo technique is used when the variables are few. The simplex search technique is used for surface involving many variables. The spiral search technique is applied when two or three variables are used [9]. A new automated technique that accelerates the inspection process by carrying out a fast registration by establishing a quick correspondence between the part to inspect and its CAD geometry is termed 'as is where is inspection' (AIWIN) is proposed in [10]. A two-step coarse registration process is proposed to provide a good initial guess for a modified ICP algorithm.

The least square method accommodates all the hard inspected points from CMM to build the mathematical geometry/shape. This paper brings a comparison of hard inspection methods (CMM) and the soft inspection procedure carried out with least square method on flatness, circularity and straightness with case study. Results of agreement are discussed with the normal distribution curves to show the reliability of the method.

2. Description

The Least Square Method is very robust in handling 'n' number of points for computation. This work focuses on the reliability on the range of tolerance achieved by the least square method for Flatness, Circularity and Straightness. The specimens considered are located on the Co-ordinate Measuring Machine (CMM) table and the points that are of interest are probed by contact method at regular intervals.

2.1. Flatness

Granite surface table is used to evaluate flatness parameter as in Fig.1. The size of the Granite surface table is 500mm X 500mm. The standard tolerance zone is 0.038 mm. The sample is divided into equal number of grids of 10mm x 10mm as shown in Fig.2. The number of sample points considered is 171.



Fig.1. Photo of Granite for surface plate

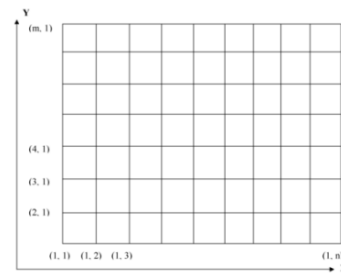


Fig.2. Grid Alignment for Flatness measurement

2.2. Circularity

Ring gauge of Ø80mm, is used to evaluate the circularity parameters as shown in Fig.3. The standard tolerance zone is 0.008 mm.



Fig.3 Ring gauge

The readings are taken on the entire surface of the ring as shown Fig.4. The number of sample points considered is 228.

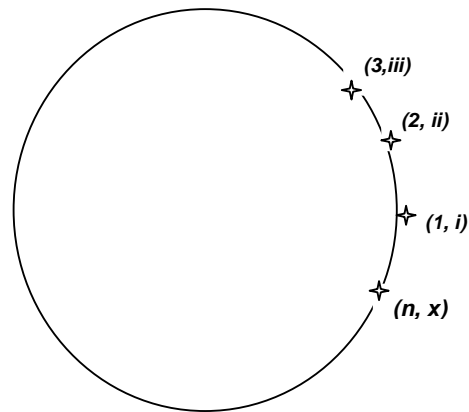


Fig.4: Circularity measurement on CMM

2.3. Straightness

The specimen used for the evaluation of the straightness is a Straight Edge Ruler of 1000mm length as shown in Fig.5. The given Standard tolerance for straightness zone is 0.026 mm.



Fig 5. Straight edge ruler

The readings are along the entire length of the ruler at spacing of about 1mm as shown in Fig.6. Number of sample points considered along the length is 1143.

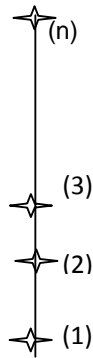


Fig. 6: Straightness measurement on CMM

2.4. Methodology

The points tracked for the three features viz., flatness, circularity and straightness on granite, ring 80 and straight edge ruler respectively form the hard inspection procedure defining the nominal value. These points are taken as input for the mathematical procedure of least square method as described in Appendix A. The Best Fit Plane/circle/straight line or the Gaussian Plane/circle/straight line is generated from the algorithms. This procedure sets up the soft inspection technique. The difference between the hard inspection and the soft inspection can be brought by analyzing the deviation between the two techniques. The deviation of the Gaussian plane to the Nominal plane is calculated by

$$d = d_0 - d_i$$

where,

- d - Actual deviation
- d_0 - Nominal value
- d_i - Inspected value for the various points
- i - i^{th} point

The actual deviation is presented in a graph for the set of points along the sampling length. The tolerance zone referred from BIS standards is termed as standard upper/lower limits (SUL/SLL). The average of maximum and minimum deviation is termed as the observed upper/lower limit (OBUL/OBLL). This margin signifies the distribution of points that shall help in inferring the shape of the bell curve, its skewness and

the bias of the deviation. This also gives an idea about the variation of best fit within these limits.

3.5. Conformance

A normal distribution bell curve is presented to support the reliability and the percentage of acceptance of soft inspection procedure to the hard inspection methods. The reliability of the specimens are calculated using the equation

$$z = \frac{x - \mu}{\sigma}, \quad \dots\dots\dots (1)$$

where,

μ - Average of deviation of points

σ - Standard deviation of the deviation of points

x - deviated points.

The probability of acceptance of the deviation of the points by soft and hard inspection procedure is presented using normal distribution curve.

$$P(z_{x=\min} < z < z_{x=\max}) \quad \dots\dots\dots (2)$$

The normal distribution curve for the all the specimens considered are drawn individually. The probability distribution function for each and every specimen is calculated using the relation.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \dots\dots\dots (3)$$

The values of the probability distribution function for the specimens are calculated at each and every point and are plotted in a graph as detailed below.

3.0. Results and Discussion

Comparative results of the flatness, circularity and the straightness for the components considered and subject of variation for each of the data sets are reported.

3.1 Flatness

With the applicability of algorithm as shown in Fig.(7) and the methodology of the Least Square, the Gaussian Plane is calculated, eqn.(4)

$$0.0231x + 0.0103y + 0.9997z = 0.0472 \quad \dots\dots (4)$$

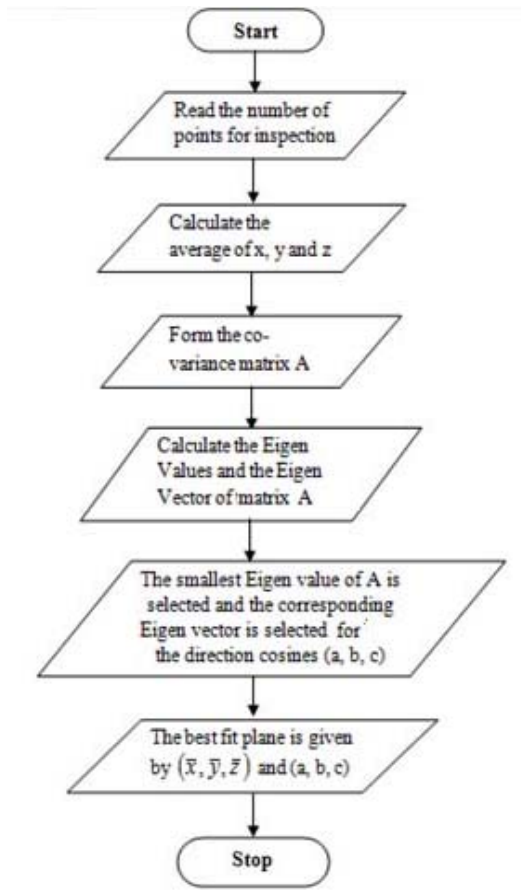


Fig.7. Flow Chart for finding the Gaussian Plane

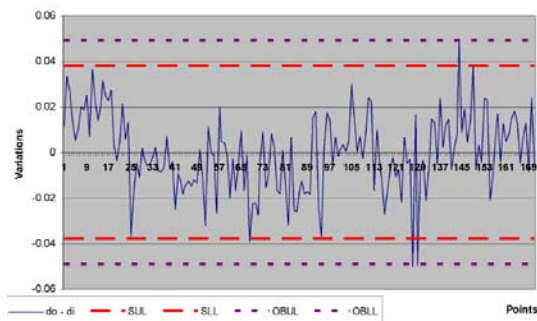


Fig.8. Graph representing the variations of the points and comparison between the standard and the obtained tolerance zone for Surface plate.

The graph depicting the variation of the distance of the points from the Gaussian plane is given in Fig.8. The tolerance zone referred in standards for Granite of this dimensions is 0.038mm. The obtained tolerance zone is 0.045 millimeters. The points that lie within the standard tolerance zone and the obtained tolerance zone are deviations that shall be controlled by best fit probabilities.

The graphs depicting the normal distribution bell curve, as in Fig.9, for the hard inspected points have resulted in $f(x)$ and best fit by least square method represented by $g(x)$ as in eqn.(3).

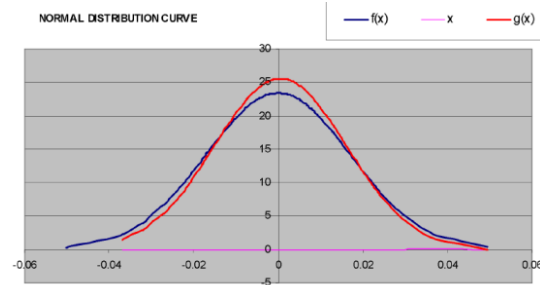


Fig.9. Normal Distribution Curve for Surface plate

The OBUL and OBLL are margins/limits to the errors. The areas where these data points lie are to be studied in detail to find whether the area plays a major influence in the acceptance of the surface of the part considered. Points lying outside this margin shall diverge the mathematical least square best fit.

There is a good agreement of $f(x)$ and $g(x)$. Variations can be attributed to the points lying outside the tolerance zone and the distance of the points from the standard and the obtained tolerance zone. Four points out of 171 points (sample space) are deviated more in the negative side which is inferred in $g(x)$.

3.2 Circularity

The mathematical equation of the circle by the least square best fit is calculated for the sample points as shown in Fig.10 and is given in eqn.(5).

$$(x - 2086.6)^2 + (y - 622.2063)^2 = (39.9958)^2 \dots(5)$$

The graph presenting the variation of the distance of the points from the Gaussian plane is given in Fig.11. The standard tolerance zone referred for Ring gauge Ø80mm is 0.008mm.

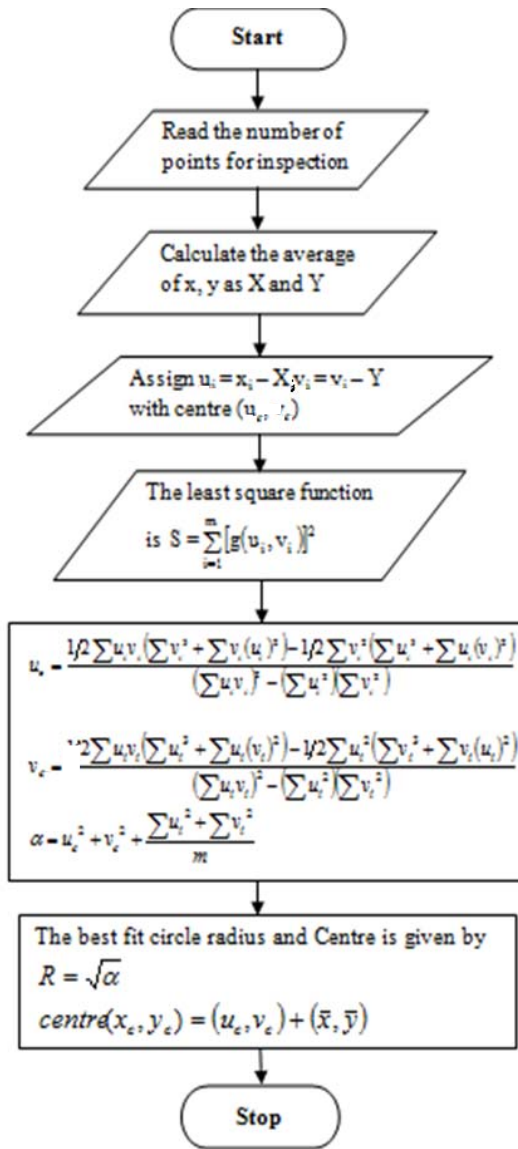


Fig.10. Flow Chart for Gaussian Circle



Fig.11. Variations of the points and comparison between the standard and obtained tolerance zone for Ring Gauge.

The obtained tolerance zone is 0.011mm. Among 228 of sample points considered for the best fit, 6 points lie outside the standard lower limit i.e., in the negative side.

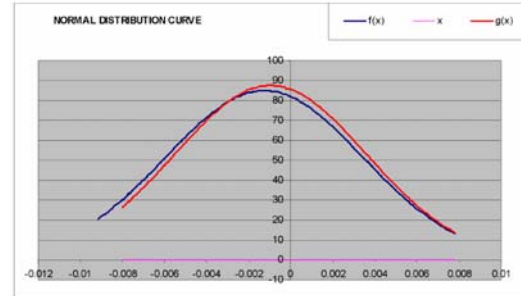


Fig.12. Normal Distribution Curve for Ring Gauge

The shift of g(x) in the left side reflects the deviations in the negative regions. Erstwhile other points are in good agreement within the tolerance zone to accept the soft inspection method as shown in Fig.12.

3.3. Straight Edge Ruler

The mathematical equation of Line for the straight edge is calculated for the sample points as in Fig.13 and is given in eqn.(6)

$$y = 559.1534 + 2.5835 \times 10^{-5} x \quad \dots\dots\dots(6)$$

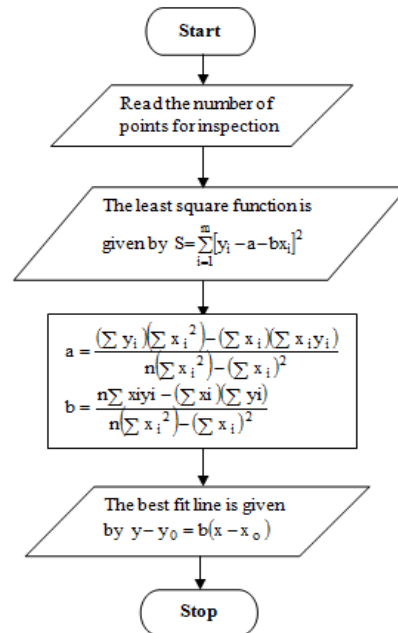


Fig.13. Flow Chart for Gaussian Line

The graph showing the variation of the distance of the points from the Gaussian plane is in Fig.14.



Fig.14. Variations of the points and comparison between the standard and the obtained tolerance zone for Straight – Edge

The standard tolerance zone referred for the straightness of length 1000mm is 0.026mm. The observed limit is 0.035mm.

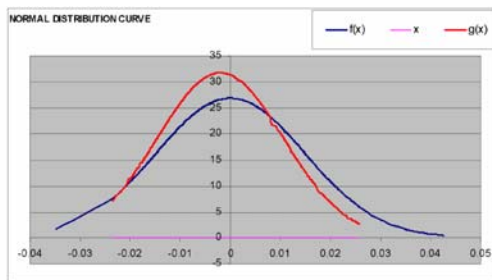


Fig.15. Graph representing the Normal Distribution Curve for Straight Edge

The areas where these data points lie are analyzed and the influence of best fit by the mathematical procedure with the hard inspection method is shown in Fig.15. 61 points out of 1143 points in the sample points are in-between the tolerance zones. Besides the good agreement, four points are classified above the observed tolerance upper limit.

4.0 Conclusions

The mathematical procedure of fitting the geometries using least square method shows a good agreement with the hard inspection technique. The normal distribution of points exhibiting the deviations along the sample length exhibits an immediate picture of best fit by least square methods. The distribution of points within the tolerance helps in achieving the sharpness of the bell curve. The density of points within the upper/lower tolerance limits reflects the skewness of the curve. The utility of least square method is helpful considering all the sample points in determining the best fit feature. The functional challenge in handling the least square method is in

negative values, which shall be attempted in the future scope of work.

5.0 Acknowledgements

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6.0 References

1. Mohib. M. N. Ahmed and Hoda. A. ElMaraghy, 2009, "Tolerance Based Localization Algorithm: Form Tolerance Verification Application", International Journal of Advanced Manufacturing and Technology.
2. Zhu Xiangyang, Han Ding and Michael. Y. Wang, 2004, "Form Error Evaluation: An Iterative Re-Weighted Least Squares Algorithm", Journal of Manufacturing Science and Engineering, Volume – 126, PP 535 - 541.
3. Huang Jyunping, 2001, "An Efficient Approach for Solving the Straightness and Flatness Problems at Large Number of Data Points", Computer Aided Design.
4. Weber Timothy, Saied Motavalli, Behrooz Fallahi and S. Hssein Cheraghi, 2001 "A Unified Approach to Form Error Evaluation", Journal of the International Societies of Precision Engineering and Nanotechnology, Volume – 26, PP 269 – 278.
5. M. H. Godallah and H. A. ElMaraghy, 1994, "A New Algorithm for Form Tolerance Evaluation", PP 286 – 291.
6. Kanada Tohru and Soichiroh Suzuki, 1993, "Evaluation of Minimum Zone Flatness by Means of Non-Linear Optimization Techniques and its Verification", Precision Engineering, PP 93 – 99.
7. M. S. Shunmugan, 1987, "New Approach for Evaluating Form Errors of Engineering Surfaces", Computer Aided Design, PP 368 – 374.
8. M. S. Shunmugam, 1987, "Comparison of Linear and Normal Deviations of Forms of Engineering Surfaces", Precision Engineering, Volume – 9, Number – 2, PP 96 – 102.
9. S. T. Huang, K. C. Fan and John. H. Wu, 1993, "A New Minimum Zone Method for Evaluating Flatness Errors", Precision Engineering, Volume – 15, Number – 1, PP 25 – 32.
10. Ravishankar. S, HNV Dutt and B.Gurumoorthy, 2012 "AIWIN"- A Fast and Jigless Inspection Technique for Machined Parts", International Journal For Advanced Manufacturing Technology, Volume-62, PP-231-240.

8.0 Appendix A

Linear Least Squares

The conventional approach for least square fit of a straight line is described below.

Consider fitting a straight line

$$y = a + bx$$

through a set of data points (x_i, y_i) , $i = 1$ to n . The minimizing function minimizes the sum of squares of the distances of the points from the straight line measured in the vertical direction. Thus

$$F = \sum_{i=1}^n (y_i - a - bx_i)^2$$

is the minimizing function. A necessary condition for F

to be minimum is $\frac{\partial f}{\partial a} = 0$ and $\frac{\partial f}{\partial b} = 0$.

Thus the partial differentiation of the above function with respect to a and b gives

$$2(-1) \sum_{i=1}^n (y_i - a - bx_i) = 0$$

$$2(-x_i) \sum_{i=1}^n (y_i - a - bx_i) = 0$$

This can be simplified as

$$\sum_{i=1}^n y_i = na + b \sum_{i=1}^n x_i$$

$$\sum_{i=1}^n x_i y_i = a \sum_{i=1}^n x_i - b \sum_{i=1}^n x_i^2$$

The equations above can be solved simultaneously to give us the values for a and b .

Normal Equation

Consider to fit a straight line, $y = a + bx$, to the set of data points $(x_1, y_1), (x_2, y_2) \dots (x_n, y_n)$. If the data points were collinear, the line would pass through n point. So

$$y_1 = a + bx_1$$

$$y_2 = a + bx_2$$

$$y_3 = a + bx_3$$

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$$y_n = a + bx_n$$

It can be written in a matrix form

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ \vdots \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$B = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ \vdots \\ \vdots \\ y_n \end{bmatrix}, A = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}, P = \begin{bmatrix} a \\ b \end{bmatrix}$$

So it can be compacted as $B = AP$

The objective vector p that minimizes the Euclidean length of the difference

$$\|B - AP\|$$

If $P = P^* = \begin{bmatrix} a^* \\ b^* \end{bmatrix}$ is a minimize vector, $y = a^* + b^*x$ is a least square straight line fit. This can be explained as

$$\|B - AP\|^2 = (y_1 - a - bx_1)^2 + (y_2 - a - bx_2)^2 + \dots + (y_n - a - bx_n)^2$$

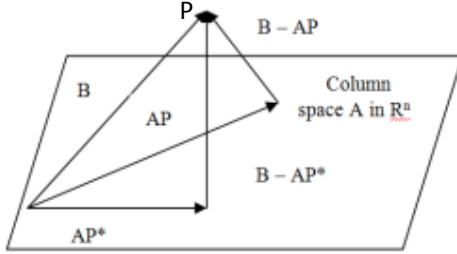
Let,

$$d_1 = (y_1 - a - bx_1)^2, d_2 = (y_2 - a - bx_2)^2, \dots, d_n = (y_n - a - bx_n)^2,$$

d can be explained as the distance from a point of a data set to fitting line.

So

$$\|B - AP\|^2 = d_1^2 + d_2^2 + \dots + d_n^2$$



To minimize $\|B - AP\|$, AP must be equal to AP* where AP* is the orthogonal projection of B on the column space of A. This implies B-AP* must be orthogonal to the column space of A. So $(B - AP^*)AP = 0$ for every vector P in R^2

This implies

$$A^T B - A^T A P^* = 0$$

$$A^T A P = A^T B$$

Which implies that P* satisfies the linear system

$$A^T A P = A^T B$$

This equation is called normal equation. This will provide the solution for P as

$$P = (A^T A)^{-1} A^T B$$

This equation can be used in the case of least square fit of a polynomial.

Eigen Vector and Singular Value Decomposition

$(A^T A)^{-1}$ is very difficult to solve. So the alternative method using singular value decomposition is used to solve P.

Singular Value Decomposition

A matrix can be decomposed in 3 matrices

$$A = USV^T$$

Where U and V are orthogonal matrices and S is a diagonal matrix containing the singular matrix of A.

Place $A = USV^T$ into normal equation

$$(USV^T)^T (USV^T) P = (USV^T)^T$$

$$B(VS^T U^T USV^T) P = VS^T U^T B$$

Knowing that

$$U^T U = I, \quad U^T = U^{-1}, \quad V^T V = I, \quad V^T = V^{-1}$$

$$\text{So } (VS^T SV^T) P = VS^T U^T B$$

Multiplying both sides by V^{-1}

$$(S^T SV^T) P = S^T U^T B$$

S is a diagonal matrix therefore $(SSV^T) P = SU^T B$

Multiplying both sides by S^{-1} two times

$$V^T P = S^{-1} U^T B$$

Again multiplying both sides by V

$$VV^T P = VS^{-1} U^T B$$

So the solution for P is

$$P = VS^{-1} U^T B$$

This equation is used in the case of least square polynomial fit.

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